COMPUTATION OF THE LONGITUDINAL TRUE-CONCENTRATION DISTRIBUTION IN VERTICAL GAS-SUSPENSION FLOWS

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Approximate theoretical relations are obtained which describe the variations of the local (averaged across the section) concentrations and the mean longitudinal concentrations of vertical monodisperse gas-suspension flows. A comparison with experimental data reveals satisfactory agreement.

Let us derive theoretical relations by means of which an estimate of the true concentration can be found and compared with experimental data [1–3].

We establish a simple relation between the longitudinal concentration distribution and the variation of particle velocity, which is defined by a steady-state flow component law, and is expressed by a formula for the throughput concentration μ. For instantaneous cross sections downstream ($\beta_l \ll 1-\beta_l$), we have

$$\frac{\beta_l}{\mu}=\frac{\rho}{\rho_s}\,\frac{v}{v_{sl}};\quad \frac{\beta_l}{\beta_{thr}}=\frac{v}{v_{sl}}=\frac{v}{v\mp v_{rl}}. \tag{1}$$

For averaged values, we have the following relations:

$$\frac{\beta}{\beta_{thr}}=\frac{\tau_s}{\tau}=\frac{v}{v_s}. \tag{2}$$

The minus sign refers to direct flow, the plus sign to counterflow.

We examine the case of an ascending direct flow, for which we use an expression which holds for particles with c_f = const (self-similar flow conditions) [14]:

$$v_{ri}=v_f\,\mathrm{cth}\gamma=v_f\,\mathrm{cth}\left(\frac{g\tau_s}{v_f}+0.5\ln\frac{v+v_f}{v-v_f}\right). \tag{3}$$

Denoting $k_v = v/v_f$, and setting $v_{ri} = v(v_{si} = 0)$, instead of (1) we get

$$\frac{\beta_l}{\beta_{thr}}=1\Bigg/\left[1-\frac{1}{k_v}\mathrm{cth}\left(\frac{g\tau_s}{v_f}+0{,}5\ln\frac{k_v+1}{k_v-1}\right)\right]=\frac{k_v}{k_v+1}\,\frac{\dfrac{k_v+1}{k_v-1}\exp\dfrac{2g\tau_s}{v_f}-1}{\exp\dfrac{2g\tau_s}{v_f}-1}. \tag{4}$$

In order to determine a relation describing the changes in concentration over the channel height, it is advantageous to substitute into (4) the path L covered by a particle for the time. Referring to an expression for τ_s [5] which holds for the condition $Z > 0.5–1$ (the error of the formula is ±3.5–8%), we get

$$Z=\frac{g}{v_f^2}(v\tau_s-L)-0{,}5\ln\frac{v_{ri}^2-v_f^2}{v_f^2},$$

$$\tau_s\simeq\frac{1}{k_v-1}\left(\frac{L}{v_f}+\frac{v_f}{g}\ln\frac{k_v+1}{2}\right). \tag{5}$$

We find

$$\frac{\beta_l}{\beta_{thr}}=\frac{k_v}{k_v+1}\times\frac{\dfrac{k_v+1}{k_v-1}\exp\left[\dfrac{2}{k_v-1}\left(\dfrac{gL}{v_f^2}+\ln\dfrac{k_v+1}{2}\right)\right]-1}{\exp\left[\dfrac{2}{k_v-1}\left(\dfrac{gL}{v_f^2}+\ln\dfrac{k_v+1}{2}\right)\right]-1}. \tag{6}$$

The value for the concentration averaged over the channel length is

$$\frac{\beta}{\beta_{thr}}=\frac{1}{L}\int\limits_0^L\frac{\beta_l}{\beta_{thr}}dL=\frac{k_v}{k_v+1}\times\left\{1+\frac{v_f^2}{gL}\ln\frac{\exp\left[\dfrac{2}{k_v-1}\left(\dfrac{gL}{v_f^2}+\ln\dfrac{k_v+1}{2}\right)\right]-1}{\exp\left(\dfrac{2}{k_v-1}\ln\dfrac{k_v+1}{2}\right)-1}\right\}. \tag{7}$$

A relation such as (7) can be obtained more simply and without error in τ_s by integrating in (7). To this end, we write the ratio of the averaged true concentration to the throughput concentration in terms of the ratio of the time of motion of the flow components, where τ_s is substituted in accordance with (5) [5]:

$$\frac{\beta}{\beta_{thr}}=\frac{\tau_s}{\tau}=\frac{k_v}{k_v-1}\left(1+\frac{v_f^2}{gL}\ln\frac{k_v+1}{2}\right). \tag{8}$$

For the counterflow, in accordance with [5], we get:

$$\tau_s\simeq\frac{1}{1-k_v}\left(\frac{L}{v_f}+\frac{v_f}{g}\ln\frac{2}{k_v+1}\right). \tag{9}$$

Then, for the averaged concentration, we have

$$\frac{\beta}{\beta_{thr}}=\frac{\tau_s}{\tau}=\frac{k_v}{1-k_v}\left(1+\frac{v_f^2}{gL}\ln\frac{2}{k_v+1}\right). \tag{10}$$

The obtained relations are valid for cases in which the hydromechanical interaction between the particles and the channel walls can be neglected. It is suggested that this interaction be taken into account via the particle-flow criterion K_W, which is the ratio of the interaction forces between the particles and the wall to the particle weight. On the basis of Pal'tsev's data [6], we have

$$K_w = \frac{mv_s^2}{gD}, \tag{11}$$

where m ≅ 0.0037.

Then, the limiting relative velocity is

$$v_{\lim.r} = v_f \sqrt{1 + K_w}, \tag{12}$$

while the initial differential equation takes the form:

$$-\frac{dv_r}{d\tau_s} = g\,\frac{v_r^2 - v_{\lim.r}^2}{v_{\lim.r}^2}. \tag{13}$$

Transforming and separating variables,

$$gd\tau_s = dv_r \Big/ \left\{ 1 - \frac{v_r^2}{v_f^2\left[1 + \frac{m}{gD}(v - v_r)^2\right]} \right\}, \tag{14}$$

we integrate (14), assuming that

$$\frac{m}{gD} \ll \frac{1}{v_f^2}; \quad \frac{2mv}{gD} \ll \frac{1}{v_f}; \quad v_{si} = 0. \tag{15}$$

Finally, we get

$$\tau_s = \frac{1}{g}\left\{ \frac{v_f}{2}\sqrt{1 + \frac{mv^2}{gD}} \times \ln\left| \frac{\frac{v_{rl}}{v_f} + \frac{mvv_f}{gD} + \sqrt{1 + \frac{mv^2}{gD}}}{\frac{v_{rl}}{v_f} + \frac{mvv_f}{gD} - \sqrt{1 + \frac{mv^2}{gD}}} \times \frac{\frac{v}{v_f} + \frac{mvv_f}{gD} - \sqrt{1 + \frac{mv^2}{gD}}}{\frac{v}{v_f} + \frac{mvv_f}{gD} + \sqrt{1 + \frac{mv^2}{gD}}} \right| - \frac{mvv_f^2}{gD} \ln \frac{\left(1 + \frac{mv^2}{gD}\right) - \frac{v_{rl}^2}{v_f^2} - \frac{2mv}{gD}v_{rl}}{1 - \frac{v^2}{v_f^2} - \frac{mv^2}{gD}} \right\}. \tag{16}$$

This expression makes it possible to estimate the time of particle motion in an ascending air stream, for the case in which the K_w criterion may not be neglected and when the relative particle velocity is known.

According to (16), the time τ_s is composed of two parts, each of which degenerates in a different fashion when $m \to 0$: the first term yields the solution previously obtained for $K_w = 0$, while the second term vanishes.

Furthermore it should be noted regarding (16) that in the general case the interaction forces between the particles and the wall can influence particle acceleration in two different ways. Thus, the additional friction forces at the channel walls act to reduce the acceleration time. This is reflected by $v_{\lim.r}$ exceeding v_f—i.e., by a decrease in the velocity imparted to the particles during the process of stabilization of their averaged motion ($v_s = v - v_{\lim.r} < v - v_f$). On the other hand, deceleration of the particle velocities in the boundary layer combined with the necessity of reaccelerating them leads to an increase in the acceleration time. This combined effect is reflected by the two terms in expression (16): the first term reflects the factors which tend to increase the time of particle motion prior to the establishment of a certain relative velocity, while the second term reflects the additional factors which tend to diminish this time, and which vanish for $K_w \to 0$.

For this reason, it is of interest to analyze expression (16) for the particular case in which the second term is much smaller than the first. A comparison of these terms shows that this case can occur for sufficiently small particles ($v_f \ll v$).

In such cases, in addition to the determination of $\tau_s = f(v_{rl})$ from formula (16), it is possible to calculate the relative velocity as a function of time or the path. Introducing, in the same manner as in (3),

$$\gamma^* = \frac{g\tau_s}{v_{\lim.r}^*} + 0.5\ln\frac{v_{ri} + v_{\lim.r}^* + \frac{mvv_f^2}{gD}}{v_{ri} - v_{\lim.r}^* + \frac{mvv_f^2}{gD}}, \tag{17}$$

where

$$v_{\lim.r}^* = v_f\sqrt{1 + \frac{mv^2}{gD}}, \tag{18}$$

from (16) (neglecting the second term) we get

$$2\gamma^* = \ln\frac{v_{rl} + v_{\lim.r}^* + \frac{mvv_f^2}{gD}}{v_{rl} - v_{\lim.r}^* + \frac{mvv_f^2}{gD}},$$

and thus

$$v_{rl} = v_{\lim.r}^* \operatorname{cth}\gamma^* - \frac{mvv_f^2}{gD}. \tag{19}$$

According to (19), the path covered by the particles during the time τ_s is:

$$L = v\tau_s - \int_0^{\tau_s} v_{ri}\,d\tau = v\tau_s\left(1 + \frac{mv_f^2}{gD}\right) - \frac{v_{\lim.r}^{*2}}{g}\ln\frac{\left[\left(v_{ri} + \frac{mvv_f^2}{gD}\right)^2 - v_{\lim.r}^{*2}\right]^{0.5}\operatorname{sh}\gamma^*}{v_{\lim.r}^*}. \tag{20}$$

In accordance with [5], we rewrite expression (20) in a form which yields the important relation $\tau_s = f(L)$: $\exp Z^* = \operatorname{sh}\gamma^*$:

$$Z^* = \frac{vg\tau}{v_{\lim.r}^{*2}}\left(1 + \frac{mv_f^2}{gD}\right) - \frac{gL}{v_{\lim.r}^{*2}} - \frac{1}{2}\ln\frac{\left(v_{ri} + \frac{mv_f^2 v}{gD}\right)^2 - v_{\lim.r}^{*2}}{v_{\lim.r}^{*2}}.$$

For $Z^* > 0.5$–1 (according to [5], the error may be assessed from $Z^* = Z$), within a certain approximation, we get:

$$\gamma^* \simeq \ln 2 + Z^*,$$

$$\tau_s = \frac{\frac{v_{\lim.r}^*}{g}\ln\frac{v_{ri} + v_{\lim.r}^* + mvv_f^2/gD}{2v_{\lim.r}^*} + \frac{L}{v_{\lim.r}^*}}{\frac{v}{v_{\lim.r}^*}\left(1 + \frac{mv_f^2}{gD}\right) - 1}. \tag{21}$$

On the basis of (19) and (21), we obtain relations for determining the local and mean true concentrations:

$$\frac{\beta_1}{\beta_{\mathrm{thr}}} = \frac{v}{v_{\mathrm{s.l}}}$$

$$= \frac{k_v}{k_v+1}\left\{\exp\left[\frac{2}{k_v\left(1+\frac{mv_{\mathrm{f}}^2}{gD}\right)-1}\times\right.\right.$$

$$\left.\times\left(\frac{gL}{v^{*2}_{\mathrm{lim.r}}}+\ln\frac{k_v+1+\frac{mvv_{\mathrm{f}}}{gD}}{2}\right)\right]+$$

$$\left.+\frac{2}{k_v+1+\frac{mvv_{\mathrm{f}}}{gD}}-1\right\}\times$$

$$\times\left\{\frac{k_v-1}{k_v+1}\exp\left[\frac{2}{k_v\left(1+\frac{mv_{\mathrm{f}}^2}{gD}\right)-1}\left(\frac{gL'}{v^{*2}_{\mathrm{lim.r}}}+\right.\right.\right.$$

$$\left.\left.+\ln\frac{k_v+1+\frac{mvv_{\mathrm{f}}}{gD}}{2}\right)\right]+$$

$$\left.\frac{2}{k_v+1+\frac{mvv_{\mathrm{f}}}{gD}}-1\right\}^{-1}, \tag{22}$$

$$\frac{\beta}{\beta_{\mathrm{thr}}} = \frac{\tau_{\mathrm{s}}}{\tau} = \frac{\frac{v^*_{\mathrm{lim.r}}v}{gL}\ln\frac{v_{\mathrm{ri}}+v^*_{\mathrm{lim.r}}+\frac{mvv_{\mathrm{f}}^2}{gD}}{2v^*_{\mathrm{lim.r}}}+\frac{v}{v^*_{\mathrm{lim.r}}}}{\frac{v}{v^*_{\mathrm{lim.r}}}\left(\frac{mv_{\mathrm{f}}^2}{gD}+1\right)-1}. \tag{23}$$

Verification of the obtained relations (19), (20), (21), (22), and (23) with respect to the limiting condition m = 0 yields expressions (3)–(5) and (6), (7) which were formally obtained for the conditions $v_{\mathrm{lim.r}}$ $v_{\mathrm{lim.r}} = v_{\mathrm{f}}$ ($K_w = 0$).

A comparison between Maksimchuk's data with respect to v_s/v and the results of computations from formula (6) is given in Fig. 1 and Table 1. The comparison is possible inasmuch as c_f = const ($Re_f \geq 10^{-3}$), $v \simeq$ const, Z > 0.5 (except the initial sections). In the calculations, use is made of the velocities imparted to a grain of wheat, which according to [1] are 10 m/sec for a grain moving in a 100-mm-diam tube, and 9.2 m/sec for a 41-mm-diam tube. The latter value was taken for channels measuring 52.5 mm in diameter. The distances to the cross sections for which the computations were performed were identical with the experimental ones. In an example, use was made of data obtained for a material throughput of 1 ton/hr, air speeds of 15, 25, and 35 m/sec, and D = 100 and 52.5 mm. According to the data in [1], a change in the mass flow rate to 4 ton/hr (μ to 10) has almost no effect on the "slipping" of the velocity components.

Table 1 shows that the deviation between the analytical and experimental data is most pronounced in the initial sections of the channel, where it varies between 12.2 and 49.5%. This can be attributed to two factors. First, the computations did not take into account the possibility that the initial particle velocity is greater than zero ($v_{ri} < v$), which depends on the conditions under which the material was introduced into the channel, and which is essential for particle acceleration. The second factor is associated with the small values of the complex Z in the initial sections; this increases the error of formula (5) from which the ratio $\beta_l/\beta_{\mathrm{thr}}$ is computed. This situation becomes the more pronounced the higher the air speed. In this connection it may be stated that the computational accuracy for such cases (Z < 0.5 to 1) may be increased by means of a relation of the type proposed in [4]. In many cases, however, such a complication of computations, which arises from the use of two analytical relations (for Z > 0.5 and Z < 0.5), one of which (for Z < 0.5) is bulky, is not justified since the discrepancy between mean concentrations is in the whole quite satisfactory (see Table 3) and does not exceed the experimental error. It should be noted that the discrepancy between the experimental data [2, 3] and an approximate relation proposed in [2] can be as high as 20–35%.

From Table 1 it can be seen that the discrepancy of data beyond the outlet section decreases, changes its sign, and increases somewhat in absolute value, amounting to only a few per cent at the end sections of the channel. One of the factors responsible for this behavior of the deviation of data is the error involved in the experimental procedure, where essentially averaged values of v_{sl} or β_l are taken as the local values in the individual cross sections.

Noteworthy, at the same time, is a certain tendency of the local concentration (and consequently of the particle velocity) toward an asymptotic approach to a constant value (Fig. 1). This characterizes, on one hand, the previously noted [4, 5] tendency of the acceleration time toward infinity, and, on the other hand, the possibility of determining approximately, with a definite accuracy, the onset of stabilized motion. The time at which the motion will stabilize is directly proportional to the velocity of the transporting medium.

Table 2 gives a comparison between the theoretical solution and experimental data obtained in [2] for a different material (quartz sand), a small channel diameter, and by a different method (β-radiation) which, however, does not yield reliable measurements of the mean concentration across the sections. Here, the same qualitative behavior of the deviations between

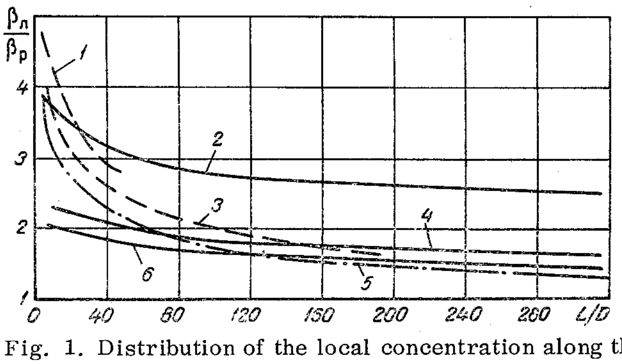

Fig. 1. Distribution of the local concentration along the channel length (D = 52.5 mm; v_f = 9.2 m/sec): curves 1, 3, 5 plotted from (6) for v = 15, 25, 35 m/sec; curves 2, 4, 6 taken from [1] for v = 15, 25, 35 m/sec.

Table 1

Comparison Between Analytical Data and Experimental Data [1]

Air speed, m/sec	Channel diameter, mm	β_l/β_{thr} at a distance from the channel inlet, m															
		0.27		0.83		1.43		2.48		3.73		5.98		10.23		16.59	
		from [1]	from (6)	from [1]	from (6)	from [1]	from (6)	from [1]	from (6)	from [1]	from (6)	from [1]	from (6)	from [1]	from (6)	from [1]	from (6)
15	100	5.0	4.39	3.74	4.01	3.23*	3.74	2.8	3.45	2.52	3.25	2.23	3.1	1.97	3.01	1.74	3.00
		(12.2)		(−7.2)		(−15.8)		—		—		—		—		—	
15	52.5	4.83	3.82	3.62	3.51	3.15*	3.25	2.74	3.00	2.48	2.83	2.20	2.68	1.92	2.62	1.70	2.58
		(20.8)		(3.04)		(−3.2)		—		—		—		—		—	
25	100	4.23	2.48	3.12	2.37	2.74	2.28	2.38	2.16	2.13	2.06	1.91	1.94	1.66*	1.80	1.47	1.72
		(41.3)		(24)		(16.8)		(9.2)		(3.3)		(−1.6)		(−8.4)		—	
25	52.5	4.10	2.33	3.05	2.23	2.66	2.14	2.31	2.03	2.08	1.93	1.86	1.82	1.62*	1.70	1.45	1.63
		(43.2)		(26.5)		(19.5)		(12.1)		(7.2)		(2.1)		(−4.9)		—	
35	100	4.06	2.05	3.04	1.95	2.65	1.94	2.30	1.85	2.07	1.80	1.85	1.70	1.61	1.58	1.43	1.50
		(49.5)		(35.9)		(26.8)		(19.5)		(13.0)		(8.1)		(1.2)		(−4.9)	
35	52.5	3.72	2.00	2.78	1.94	2.44	1.90	2.13	1.82	1.91	1.74	1.70	1.64	1.48	1.54	1.32	1.46
		(46.2)		(30.0)		(22.0)		(14.5)		(8.9)		(3.5)		(−4.0)		(−10.6)	

Note: 1) Experimental data which follow values marked with an asterisk no longer satisfy the condition $v - v_s = v_f$; 2) the relative deviations between analytical and experimental values of β_l/β_{thr}, in percent, are given in parentheses.

Table 2

Comparison with the Experimental Data in [2]

Distance from the inlet, m	Ratio of the local to the throughput concentration, β_l/β_{thr}: from [2]	Ratio of the local to the throughput concentration, β_l/β_{thr}: from formula (6)	Relative deviation, %	Remarks
0.12	8.29	4.82	41.7	1) Quartz and particles: $d_s = 0.67$ mm, $v_f = 4.85$ m/sec, $v = 6.8$ m/sec, D = 21.5 mm. 2) Coefficients in the formula [2], constant over the entire channel length: $a = 9.151$, $b = 0.484 \cdot 10^{-2}$, $c = 4.452$.
0.21	6.45	4.52	30.0	
0.308	5.44	4.29	21.2	
0.412	4.91	4.10	16.5	
0.612	4.56	3.87	15.1	
0.712	4.50	3.80	15.5	
0.91	4.47	3.70	17.2	
1.227	4.45	3.60	19.1	
1.625	4.45	3.54	20.4	
2.020	4.45	3.46	22.2	

the analytical and experimental data as in the previous case (larger deviations at the inlet and outlet sections) can be observed. In addition, we note that in [2] the experimental local values of β_l/β_{thr} are not given, so that these values were assessed for Table 2 from a relation in [2], having its own approximation error, on the basis of the coefficients a, b, c which are constant for the entire channel, and which are given for the series of experiments examined in [2].

According to [1], the mean values of the concentration (velocity) along the channel are

$$v_s = 0.8 v_{s1}; \quad \frac{\beta}{\beta_{thr}} = \frac{v}{v_s} = 1.25 \frac{v}{v_{s1}}. \tag{24}$$

Values for β/β_{thr} are given in [2, 3]. For calculating β/β_{thr} we use relations (7) and (8). The results of a comparison between the experimental and computed values of the ratio of the mean true concentration to the throughput concentration are given in Table 3. For all cases in which experimental data yield an irregular inequality $v_r < v_f < v_{lim.\,r}$, averaging is performed only over the channel lengths where this inequality does not take place. The need for this approach occurred only for few experimental data [1]. In addition, Fig. 2 shows a comparison between the theoretical and experimental data [1] for some most illustrative cases (for the lowest air speed of 15 m/sec) of averaging the true concentration over the channel length (starting from the inlet), including sections where the condition is fulfilled and not fulfilled. From the figure, it can be seen that for sections with $v_r < v_f$, the data deviation changes its sign while its absolute

Table 3
Comparison Between the Analytical and Experimental [1–3] Mean-Concentration Data

Initial experimental data	Particle size, mm	Imparted velocity, m/sec	Air speed, m/sec	Ratio of the mean true concentration to the throughput concentration β/β_{thr}			Deviation, %	
				experimental	from (7)	from (8)	with respect to (7)	with respect to (8)
Data of [1]; L = 16.5 m; grain; D = 100 mm	4—4.5	10	15	4.06	4.12	3.42	1.48	15.7
		10	25	1.84	1.92	2.25	4.35	22.2
		10	35	1.64	1.68	2.14	2.44	30.4
Same for D = 52.5 mm	»	9.2	15	3.42	3.38	2.97	1.17	13.2
		9.2	25	1.82	1.82	2.09	0	14.7
		9.2	35	1.65	1.45	1.97	12.1	21.2
Data of [2]; L = 2.3 m; quartz sand; D = 21.5 mm	0.28	2.25	6.25	2.14	1.15/1.86	1.66/1.96	13.1	8.4
	0.67	4.85	6.80	4.93	3.70	4.17	24.9	15.4
	0.67	4.85	10.17	3.60	2.26/2.68	2.78/3.18	25.6	11.7
	1.26	6.7	12.70	5.21	2.26/3.34	3.65/4.19	35.9	19.5
Data [3]; L = 1.7 m; millet; D = 26 mm	2.4	6.4	13.50	3.76	2.50/3.02	3.98	19.6	5.8
	2.4	6.4	14.90	4.28	2.33/2.98	3.96	30.4	7.5
	2.4	6.4	16.80	3.51	2.16/2.85	3.96	18.8	11.4
	2.4	6.4	21.0	4.02	1.96/2.85	4.14	29.2	3.0

Note: The values given in the denominator of a fraction refer to β/β_{thr} values obtained with allowance for the influence of K_W after one iteration step.

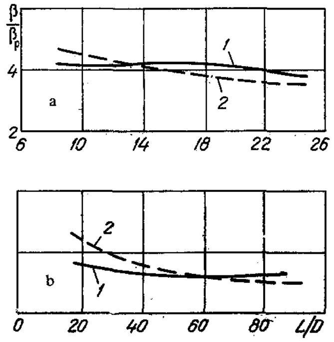

Fig. 2. Longitudinal distribution of lengthwise averaged concentration: a) D = 100 mm, v_f = 10 m/sec, v = = 15 m/sec; b) D = 52.5 mm, v = 9.2 m/sec; 1) formula (7); 2) taken from [1].

value is hardly affected. The influence of the criterion K_W can be assessed approximately by the following iterative technique. After determining β/β_{thr} from formula (7) or (8), v_s and $K_W = mv_s^2/gD$ are evaluated. Then, $v_{lim.r} = v_f(1 + K_W)^{1/2}$ is substituted for v_f in the analytical relation; in most cases, this is sufficient for obtaining preliminary estimates of the influence of K_W on β/β_{thr}.

From Table 3, the following may be established: a) the deviation between results obtained by the computational method proposed and experimental data obtained by different methods by various investigators for different channel and particle dimensions and different particle velocities lies between 0 and 35%; the mean value of the deviation does not exceed 15 to 20%; b) the data obtained in [1] are best approximated by formula (7) (mean deviation on the order of 4%), data [2] are best approximated by formula (8) (mean deviation on the order of 12%); formula (8) yields also the best approximation for data [3] (mean deviation on the order of 7%); c) allowance for the influence of K_W by an approximate method (based on the substitution of the relative limiting velocity for v_f in formulas for β/β_{thr}), in many cases, does not improve the discrepancy between the values compared. This should not be attributed exclusively to the imperfection of the computational relations, since the specific features and errors of the various methods employed in [1–3] may also be involved.

NOTATION

β is the volume concentration, m^3/m^3; v is the absolute velocity, m/sec; β_{thr} is the throughput volume concentration, $m^3 \cdot sec/m^3 \cdot sec$; v_r, $v_{lim.r}$, and v_f are the relative velocity, limiting relative velocity, and free-fall velocity, respectively; D is the channel diameter; L is the path length; ρ is the density; τ is the time of motion. Subscripts: s refers to a solid particle; values without this subscript refer to a continuous medium; subscript l refers to local values averaged across the section; values without this subscript are averaged along the length; i refers to the initial instant.

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